Math 240 Quiz 6 (5.1-5.3)

NetID: _____ **Class time:** _____

Instructions: Calculators, course notes and textbooks are **NOT** allowed on the quiz. All numerical answers **MUST** be exact; e.g., you should write π instead of 3.14..., $\sqrt{2}$ instead of 1.414..., and $\frac{1}{3}$ instead of 0.3333... Explain your reasoning using complete sentences and correct grammar, spelling, and punctuation.
Show ALL of your work!
You have 20 minutes.

Question 1 (3 points). Give examples.

(i) A nonzero 2×2 matrix with zero as its only eigenvalue

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

(ii) A 2×2 matrix that is diagonalizable but not invertible

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(iii) A 2×2 matrix that is neither diagonalizable nor invertible

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Question 2 (6 points). Given

$$A = \begin{bmatrix} 0 & 4 & 6 \\ 1 & 0 & 3 \\ -1 & -2 & -5 \end{bmatrix}$$

find matrices B and C, with B invertible and C diagonal, such that $AB = BC$.

$$\det(A-\lambda I) = \begin{vmatrix} -\lambda & 4 & 6 \\ 1 & -\lambda & 3 \\ -1 & -2 & -5-\lambda \end{vmatrix}$$

$$= -\lambda \begin{vmatrix} -\lambda & 3 \\ -2 & -5-\lambda \end{vmatrix} - 4 \begin{vmatrix} 1 & 3 \\ -1 & -5-\lambda \end{vmatrix} + 6 \begin{vmatrix} 1 & -\lambda \\ -1 & -2 \end{vmatrix}$$

$$= -\lambda(\lambda^2+5\lambda+6) - 4(-\lambda-2) + 6(-2-\lambda)$$

$$= -\lambda(\lambda+2)(\lambda+3) + (\lambda+2)4 + (\lambda+2)(-6)$$

$$= (\lambda+2)(-\lambda^2-3\lambda-2)$$

$$= -(\lambda+2)^2(\lambda+1)$$

$\underline{\lambda = -2}$ $\quad A-\lambda I = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \vec{x} = \begin{bmatrix} -2s-3t \\ s \\ t \end{bmatrix} = s\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$

$\underline{\lambda = -1}$ $\quad A-\lambda I = \begin{bmatrix} 1 & 4 & 6 \\ 1 & 1 & 3 \\ -1 & -2 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & 6 \\ 0 & -3 & -3 \\ 0 & 2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & 6 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \vec{x} = t\begin{bmatrix} -10 \\ -1 \\ 1 \end{bmatrix}$

Thus $B = \begin{bmatrix} -2 & -3 & -10 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Question 3 (1 point). Find a second identity $AB = BC$ with the same A and C above, but with a different invertible matrix B.

Take $B = \begin{bmatrix} 2 & -3 & -10 \\ -1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$ instead.